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SOLVABILITY OF INTEGRAL EQUATIONS VIA THE TECHNIQUE OF MEASURES OF NONCOMPACTNESS

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Abstract. Motivated by the work of [Mohammadi et al., Mathematics, 2019, 7, 575.], we extend here Darbo's fixed point theorem in a Banach space using the combined technique of Wardowski-Mizoguchi-Takahashi contraction. The existence of solution for a system of integral equations is provided and an example to support the effectiveness of our results is also given. Our results generalize and extend some famous comparable obtained results.

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1. Introduction and preliminaries

The first "measure of non-compactness (MNC, for short)" was defined by Kuratowski [12], that is, for any bounded set Ω in a metric space, it is the infimum of all the numbers $\varepsilon > 0$ such that Ω can be covered by a finite number of sets with diameters $< \varepsilon$. The Hausdorff measure is another MNC, which is defined as

$$\mu(\Omega) = \inf\{\varepsilon > 0 : \Omega \subset \cup_{i=1}^{n} B(x_i, r_i) : x_i \in \Omega, r_i < \varepsilon\,(i = 1, 2, \ldots, n), n \in \mathbb{N}\}.$$

In the study of infinite systems of differential equations (ISDEs), the MNC plays a signifying role. In recent times, the MNC has been effectively applied in sequence spaces and function spaces for variant classes of differential equations. One of the most applicable results in organizing the existence of solutions of differential equations, integral equations and integro-differential equations is Darbo's fixed point theorem [10]. The controllability problem of dynamical systems represented by implicit differential equations (see [3]) is another field in which Darbo's fixed-point theorem can be used. Schauder's fixed point principle [1] and Darbo's fixed point theorem [10] are two important results in this study.

For more details on applications of MNC, we refer the reader to [1, 2, 4–8, 11, 14, 15, 17].

In this paper, first we generalize the Darbo's fixed point theorem in a Banach space via Wardowski-Mizoguchi-Takahashi combined contraction and secondly we study the existence of solution for the following system of integral equations:

$$\begin{cases} \rho(\iota) = f\Big(\iota, h(\iota, \rho(\theta(\iota)), \sigma(\theta(\iota))), \int_0^{\zeta(\iota)} g\Big(\iota, \kappa, \rho(\theta(\kappa)), \sigma(\theta(\kappa))\Big) d\kappa\Big) \\ \sigma(\iota) = f\Big(\iota, h(\iota, \sigma(\theta(\iota)), \rho(\theta(\iota))), \int_0^{\zeta(\iota)} g\Big(\iota, \kappa, \sigma(\theta(\kappa)), \rho(\theta(\kappa))\Big) d\kappa\Big) \end{cases} \tag{1.1}$$

where $\iota \in [0, T]$.

We gather several concepts which are used throughout the text. We denote by $\mathbb{R}$ the set of real numbers, and $\mathbb{R}_+ = [0, +\infty)$. Let $(\mathfrak{B}, \|\cdot\|)$ denote a real Banach space. In addition, let $\overline{B}(\rho, r)$ be the closed ball with center ρ and radius r and $\overline{B}_r := \overline{B}(0, r)$. For a nonempty subset Q of $\mathfrak{B}$, let $\overline{Q}$ and $Conv(Q)$ denote the closure and the closed convex hull of Q, respectively. Let $\mathcal{NB}_{\mathfrak{B}}$ be the collection of all nonempty bounded subsets of $\mathfrak{B}$ and $\mathcal{RC}_{\mathfrak{B}}$ be the family of all relatively compact subsets of $\mathfrak{B}$.

Definition 1 ([8])**.** $\chi\colon \mathcal{NB}_{\mathfrak{B}} \longrightarrow \mathbb{R}_+$ is called a "measure of noncompactness" in $\mathfrak{B}$ if:

1° $ker\chi = \{Q \in \mathcal{NB}_{\mathfrak{B}} : \chi(Q) = 0\}$ is nonempty and $ker\chi \subset \mathcal{RC}_{\mathfrak{B}}$;
2° $Q \subset \Delta \Longrightarrow \chi(Q) \leq \chi(\Delta)$;
3° $\chi(\overline{Q}) = \chi(Q)$;
4° $\chi(Conv(Q)) = \chi(Q)$;
5° $\chi(\lambda Q + (1-\lambda)\Delta) \leq \lambda\chi(Q) + (1-\lambda)\chi(\Delta)$ for all $\lambda \in [0,1]$;
6° for all sequence $\{Q_n\}$ of closed sets in $\mathcal{NB}_{\mathfrak{B}}$ with the reservations $Q_{n+1} \subset Q_n$ for all $n = 1, 2, 3, \cdots$, and $\lim_{n\to\infty} \chi(Q_n) = 0$, then $Q_\infty = \cap_{n=1}^{\infty} Q_n \neq \varnothing$.

2. MAIN RESULTS

Recall the Mizoguchi-Takahashi mapping.

Definition 2 ([13])**.** $Ł\colon (0,\infty) \longrightarrow [0,1)$ with the restriction $\limsup_{\omega \longrightarrow t^+} Ł(\omega) < 1$, for any $t \geq 0$ is called a Mizoguchi-Takahashi (MT) mapping. We denote this class by $\mathcal{M} - \mathcal{T}$.

Let Π be the collection of all maps $\pi\colon [0,\infty) \longrightarrow [0,\infty)$ so that

(1) $\pi(\eta_n) \to 0$ if and only if $\eta_n \to 0$ for any sequence $\{\eta_n\}$ in $[0,\infty)$;
(2) π is a nondecreasing mapping.

In this paper, some enlargements of Darbo's fixed point theorem via Wardowski type Mizoguchi-Takahashi's contractions have been obtained. Moreover, we provide an application for a system of functional integral equations.

Let $\mathbb{A}$ be the family of all functions $\Gamma\colon (0,\infty) \to \mathbb{R}$ so that:

(Γ_1) Γ is continuous and increasing;

(Γ_2) $\lim_{n\to\infty} \iota_n = 1$ iff $\lim_{n\to\infty} \Gamma(\iota_n) = 0$ for all $\{\iota_n\} \subseteq (0,\infty)$.

Note that from (Γ_1) and (Γ_2), we have $\Gamma(1) = 0$. Some examples of elements in $\mathbb{A}$ are as follows:

(i) $\Gamma_1(\iota) = \ln(\iota)$,

(ii) $\Gamma_2(\iota) = \ln(\iota - \frac{1}{\iota} + 1)$,

(iii) $\Gamma_3(\iota) = -\frac{1}{\sqrt{\iota}} + 1$,

(iv) $\Gamma_5(\iota) = -\frac{1}{\iota} + \iota$.

Here and afterwards we take C as a nonempty, bounded, closed and convex subset of a Banach space $\mathfrak{B}$ and $\mathcal{G}: C \to C$ a continuous operator.

Theorem 1. *Let*

$$\Gamma(\pi(\chi(\mathcal{G}(Q)))) \leq \Gamma(Ł(\pi(\chi(Q)))) + \Gamma(\pi(\chi(Q)))$$

for all $Q \subseteq C$ *with* $\chi(Q) \neq 0$ *and* $\chi(\mathcal{G}(Q)) \neq 0$ *where* $\Gamma \in \mathbb{A}$, $Ł \in \mathcal{M} - \mathcal{T}$, $\pi \in \Pi$ *and* χ *is an arbitrary MNC. Then* $\mathcal{G}$ *has at least one fixed point in* C.

Proof. Let $\{C_n\}$ be defined by $C_0 = C$ and $C_{n+1} = \overline{Conv}(\mathcal{G}(C_n))$ for all $n \in \mathbb{N}$.

Assume that there exists an integer $N \in \mathbb{N}$ such that $\chi(C_N) = 0$. Then C_N is relatively compact and so according to the Schauder Theorem $\mathcal{G}$ admits a fixed point. So, let $\chi(C_n) > 0$ for each $n \in \mathbb{N}$.

Obviously, $\{C_n\}_{n\in\mathbb{N}}$ is a sequence of nonempty, bounded, closed and convex sets such that

$$C_0 \supseteq C_1 \supseteq \cdots \supseteq C_n \supseteq C_{n+1}.$$

On the other hand

$$\Gamma(\pi(\chi(C_{n+1}))) = \Gamma(\pi(\chi(\mathcal{G}(C_n)))) \leq \Gamma(Ł(\pi(\chi(C_n)))) + \Gamma(\pi(\chi(C_n))), \tag{2.1}$$

for each $n \in \mathbb{N}$.

According to the properties of measure of noncompactness χ and the properties of function π, $\{\pi(\chi(C_n)\}$ is a positive decreasing and bounded below sequence of real numbers.

Thus, $\{\pi(\chi(C_n)\}_{n\in\mathbb{N}}$ is a convergent sequence. Suppose that $\lim_{n\to\infty} \pi(\chi(C_n)) = r$.

We prove that $r = 0$. Suppose to the contrary that $r > 0$. Taking the upper limit in (2.1) when $n \to \infty$, we obtain that

$$\Gamma(r) \leq \Gamma(\limsup_{n\to\infty} Ł(\pi(\chi(C_n)))) + \Gamma(r) < \Gamma(r),$$

which is a contradiction. So, $\lim_{n\to\infty} \pi(\chi(C_n)) = r = 0$.

Therefore $\lim_{n\to\infty} \chi(C_n) = 0$. According to principle (6°) of Definition 1 we derive that the set $C_\infty = \bigcap_{n=1}^{\infty} C_n$ is a nonempty, closed and convex set and it is invariant under $\mathcal{G}$ and belongs to $Ker\chi$. Then, in the light of the Schauder Theorem $\mathcal{G}$ possesses a fixed point. □

Taking $\Gamma(\iota) = \ln(\iota)$ in Theorem 1, we have

Corollary 1. *Let*

$$\pi(\chi(\mathcal{G}(Q))) \leq Ł(\pi(\chi(Q)))\pi(\chi(Q)) \tag{2.2}$$

where $Q \subseteq C$ with $\chi(Q) \neq 0$ and $\chi(\mathcal{G}(Q)) \neq 0$, $Ł \in \mathcal{M} - \mathcal{T}$, $\pi \in \Pi$ and χ is an arbitrary MNC. Then $\mathcal{G}$ possesses at least one fixed point in C.

Taking $\Gamma(\iota) = -\frac{1}{\sqrt{\iota}} + 1$ and if π be the identity function in Theorem 1, we have:

Corollary 2. *Let*

$$\chi(\mathcal{G}(Q)) \leq \frac{Ł(\chi(Q))\chi(Q)}{(\sqrt{Ł(\chi(Q))} + \sqrt{\chi(Q)} - \sqrt{\chi(Q)Ł(\chi(Q))})^2} \tag{2.3}$$

where $Q \subseteq C$ with $\chi(Q) \neq 0$ and $\chi(\mathcal{G}(Q)) \neq 0$, $Ł \in \mathcal{M} - \mathcal{T}$ and χ is an arbitrary MNC. Then $\mathcal{G}$ has at least one fixed point in C.

3. COUPLED FIXED POINT

Definition 3 ([9]). The ordered pair $(\rho, \sigma) \in \mathfrak{B}^2$ is called a coupled fixed point of a mapping $\mathcal{G} : \mathfrak{B} \times \mathfrak{B} \to \mathfrak{B}$ if $\mathcal{G}(\rho, \sigma) = \rho$ and $\mathcal{G}(\sigma, \rho) = \sigma$.

Theorem 2 ([8]). *Let $\chi_1, \chi_2, \ldots, \chi_n$ be measures of noncompactness in Banach spaces $\mathfrak{B}_1, \mathfrak{B}_2, \ldots, \mathfrak{B}_n$, respectively, and let $\mathcal{G} \colon [0,\infty)^n \longrightarrow [0,\infty)$ be a convex mapping with the condition $\mathcal{G}(\rho_1, \ldots, \rho_n) = 0$ if and only if $\rho_i = 0$ for all $i = 1, 2, \ldots, n$. Then*

$$\tilde{\chi}(Q) = \mathcal{G}(\chi_1(Q_1), \chi_2(Q_2), \ldots, \chi_n(Q_n)),$$

is a measure of noncompactness in $\mathfrak{B}_1 \times \mathfrak{B}_2 \times \ldots \times \mathfrak{B}_n$, where Q_i is the natural projection of Q into $\mathfrak{B}_i$, for all $i = 1, 2, \ldots, n$.

Now, let Γ be a subadditive mapping. As in the previous section, let C be a nonempty, bounded, closed and convex subset of a Banach space $\mathfrak{B}$ and $\mathcal{G} \colon C \times C \to C$ be a continuous function. Then we have the following results.

Theorem 3. *Let*

$$\Gamma\Big(\pi\big(\chi(\mathcal{G}(Q_1 \times Q_2))\big)\Big) \leq \frac{1}{2}\Big[\Gamma\big(Ł(\pi(\chi(Q_1) + \chi(Q_2)))\big) + \Gamma\big(\pi(\chi(Q_1) + \chi(Q_2))\big)\Big] \tag{3.1}$$

for all subsets Q_1, Q_2 of C with $\chi(\mathcal{G}(Q_1 \times Q_2)) \neq 0$ and $\chi(Q_1)$ or $\chi(Q_2) \neq 0$, where $\Gamma \in \mathbb{A}$, $Ł \in \mathcal{M} - \mathcal{T}$, $\pi \in \Pi$ and χ is an arbitrary MNC. Then $\mathcal{G}$ has at least a coupled fixed point.

Proof. Define $\widetilde{\mathcal{G}} \colon C^2 \to C^2$ by

$$\widetilde{\mathcal{G}}(\rho, \sigma) = (\mathcal{G}(\rho, \sigma), \mathcal{G}(\sigma, \rho)).$$

Clearly, $\widetilde{\mathcal{G}}$ is continuous.

Let $\mathcal{Q} \subset C^2$ be a nonempty subset. Evidently, $\widetilde{\chi}(\mathcal{Q}) = \chi(\mathcal{Q}_1) + \chi(\mathcal{Q}_2)$ is a (MNC) [8], where $\mathcal{Q}_1$ and $\mathcal{Q}_2$ are the natural projections of $\mathcal{Q}$ into $\mathfrak{B}$. From (3.1) we have

$$\begin{aligned}
\Gamma\Big(\pi(\widetilde{\chi}(\widetilde{\mathcal{G}}(\mathcal{Q})))\Big) &= \Gamma\Big(\pi(\widetilde{\chi}(\mathcal{G}(\mathcal{Q}_1 \times \mathcal{Q}_2) \times \mathcal{G}(\mathcal{Q}_2 \times \mathcal{Q}_1)))\Big) \\
&= \Gamma\Big(\pi(\chi(\mathcal{G}(\mathcal{Q}_1 \times \mathcal{Q}_2)) + \chi(\mathcal{G}(\mathcal{Q}_2 \times \mathcal{Q}_1)))\Big) \\
&\leq \frac{1}{2}\Big[\Gamma\big(Ł(\pi(\chi(\mathcal{Q}_1) + \chi(\mathcal{Q}_2)))\big) + \Gamma\big(\pi(\chi(\mathcal{Q}_1) + \chi(\mathcal{Q}_2))\big)\Big] \\
&\quad + \frac{1}{2}\Big[\Gamma\big(Ł(\pi(\chi(\mathcal{Q}_1) + \chi(\mathcal{Q}_2)))\big) + \Gamma\big(\pi(\chi(\mathcal{Q}_1) + \chi(\mathcal{Q}_2))\big)\Big] \\
&\leq \Gamma\Big(Ł(\pi(\chi(\mathcal{Q}_1) + \chi(\mathcal{Q}_2)))\Big) + \Gamma\Big(\pi(\chi(\mathcal{Q}_1) + \chi(\mathcal{Q}_2))\Big) \\
&= \Gamma\Big(Ł(\pi(\widetilde{\chi}(\mathcal{Q})))\Big) + \Gamma\Big(\pi(\widetilde{\chi}(\mathcal{Q}))\Big).
\end{aligned}$$

Now, according to Theorem 1 we derive that $\widetilde{\mathcal{G}}$ admits at least a fixed point which yields that $\mathcal{G}$ possesses at least a coupled fixed point. □

Theorem 4. *Suppose that*

$$\Gamma\Big(\pi\big(\chi(\mathcal{G}(\mathcal{Q}_1 \times \mathcal{Q}_2))\big)\Big) \leq \Gamma\big(Ł(\pi(\max\{\chi(\mathcal{Q}_1), \chi(\mathcal{Q}_2)\}))\big) + \Gamma\big(\pi(\max\{\chi(\mathcal{Q}_1), \chi(\mathcal{Q}_2)\})\big) \tag{3.2}$$

for all subsets $\mathcal{Q}_1, \mathcal{Q}_2$ of C with $\chi(\mathcal{G}(\mathcal{Q}_1 \times \mathcal{Q}_2)) \neq 0$ and $\chi(\mathcal{Q}_1)$ or $\chi(\mathcal{Q}_2) \neq 0$, where $\Gamma \in \mathbb{A}$ is a mapping, $Ł \in \mathcal{M} - \mathcal{T}$, $\pi \in \Pi$ is also a mapping and χ is an arbitrary MNC. Then $\mathcal{G}$ has at least a coupled fixed point.

Proof. Define the mapping $\widetilde{\mathcal{G}} : C^2 \to C^2$ by

$$\widetilde{\mathcal{G}}(\rho, \sigma) = (\mathcal{G}(\rho, \sigma), \mathcal{G}(\sigma, \rho)).$$

It is clear that $\widetilde{\mathcal{G}}$ is continuous. Clearly, $\widetilde{\chi}(\mathcal{Q}) = \max\{\chi(\mathcal{Q}_1), \chi(\mathcal{Q}_2)\}$ is an MNC, where $\mathcal{Q}_1$ and $\mathcal{Q}_2$ denote the natural projections of $\mathcal{Q}$ into $\mathfrak{B}$. Let $\mathcal{Q} \subset C^2$ be a nonempty subset. From (3.2) we obtain that

$$\begin{aligned}
\Gamma\Big(\pi(\widetilde{\chi}(\widetilde{\mathcal{G}}(\mathcal{Q})))\Big) &= \Gamma\Big(\pi(\widetilde{\chi}(\mathcal{G}(\mathcal{Q}_1 \times \mathcal{Q}_2) \times \mathcal{G}(\mathcal{Q}_2 \times \mathcal{Q}_1)))\Big) \\
&= \Gamma\Big(\pi(\max\{\chi(\mathcal{G}(\mathcal{Q}_1 \times \mathcal{Q}_2)), \chi(\mathcal{G}(\mathcal{Q}_2 \times \mathcal{Q}_1))\})\Big) \\
&= \max\Big\{\Gamma\Big(\pi(\chi(\mathcal{G}(\mathcal{Q}_1 \times \mathcal{Q}_2)))\Big), \Gamma\Big(\pi(\chi(\mathcal{G}(\mathcal{Q}_2 \times \mathcal{Q}_1)))\Big)\Big\} \\
&\leq \Gamma\Big(Ł(\pi(\max\{\chi(\mathcal{Q}_1), \chi(\mathcal{Q}_2)\}))\Big) + \Gamma\Big(\pi(\max\{\chi(\mathcal{Q}_1), \chi(\mathcal{Q}_2)\})\Big) \\
&= \Gamma\Big(Ł(\pi(\widetilde{\chi}(\mathcal{Q})))\Big) + \Gamma\Big(\pi(\widetilde{\chi}(\mathcal{Q}))\Big).
\end{aligned}$$

Theorem 1 deduces that $\widetilde{\mathcal{G}}$ has at least a fixed point and so, $\mathcal{G}$ has at least a coupled fixed point. □

4. Application

Now, we study the existence of solutions for the following system of equations:

$$\begin{cases} \rho(\iota) = f\Big(\iota, h(\iota, \rho(\theta(\iota)), \sigma(\theta(\iota))), \int_0^{\zeta(\iota)} g\Big(\iota, \kappa, \rho(\theta(\kappa)), \sigma(\theta(\kappa))\Big) d\kappa\Big) \\ \sigma(\iota) = f\Big(\iota, h(\iota, \sigma(\theta(\iota)), \rho(\theta(\iota))), \int_0^{\zeta(\iota)} g\Big(\iota, \kappa, \sigma(\theta(\kappa)), \rho(\theta(\kappa))\Big) d\kappa\Big) \end{cases} \tag{4.1}$$

in the space $C([0,T])$ consisting of all bounded and continuous real functions ρ on $[0,T]$ with $\|\rho\| = \sup\{|\rho(\iota)| : 0 \leq \iota \leq T\}$.

The modulus of the continuity of $\rho \in C([0,T])$ is defined by

$$\omega(\rho, \varepsilon) = \sup\{|\rho(\iota) - \rho(\kappa)| : \iota, \kappa \in [0,T], |\iota - \kappa| \leq \varepsilon\}.$$

Let

$$\omega(Q, \varepsilon) = \sup\{\omega(\rho, \varepsilon) : \rho \in Q\},$$

$$\omega_0(Q) = \lim_{\varepsilon \to 0} \omega(Q, \varepsilon).$$

It is well known that the above function is a measure of noncompactness in the space $Q = C([0,T])$.

Theorem 5. *Suppose that:*

(i) $\theta, \zeta : [0,T] \longrightarrow [0,T]$ *are continuous functions,*

(ii) *The functions* $f, h : [0,T] \times \mathbb{R}^2 \longrightarrow \mathbb{R}$ *are continuous and*

$$\Gamma(|f(\iota, \rho_1, \rho_2) - f(\iota, \sigma_1, \sigma_2)|) \leq \Gamma(Ł(\max\{|\rho_1 - \sigma_1|, |\rho_2 - \sigma_2|\})) + \Gamma(\max\{|\rho_1 - \sigma_1|, |\rho_2 - \sigma_2|\})$$

and

$$|h(\iota_1, \rho_1, \rho_2) - h(\iota_2, \sigma_1, \sigma_2)| \leq \max\{|\iota_1 - \iota_2|, |\rho_1 - \sigma_1|, |\rho_2 - \sigma_2|\},$$

for all $\iota_1, \iota_2 \in [0,T]$ *and* $\rho_1, \rho_2, \sigma_1, \sigma_2 \in \mathbb{R}$,

(iii) $M := \sup\{\big|f(\iota, 0, 0)\big| : \iota \in [0,T]\}$ *and* $N := \sup\{\big|h(\iota, 0, 0)\big| : \iota \in [0,T]\}$,

(iv) $g : [0,T] \times [0,T] \times \mathbb{R}^2 \longrightarrow \mathbb{R}$ *is bounded and continuous and*

$$G := \sup\Big\{\Big|\int_0^{\zeta(\iota)} g(\iota, \kappa, \rho(\theta(\kappa)), \sigma(\theta(\kappa))) d\kappa\Big| : \iota, \kappa \in [0,T], \rho, \sigma, \in C([0,T])\Big\},$$

(v) *There exists a positive solution* r_0 *to the inequality*

$$\Gamma^{-1}[\Gamma(Ł(\max\{r + N, G\})) + \Gamma(\max\{r + N, G\})] + M \leq r.$$

Then the system of integral equations (4.1) *has at least one solution in the space* $(C([0,T]))^2$.

Proof. Let

$$\mathcal{G}\colon C([0,T])\times C([0,T])\longrightarrow C([0,T])$$

be defined by

$$\mathcal{G}(\rho,\sigma)(\iota)=f\Big(\iota,h(\iota,\rho(\theta(\iota)),\sigma(\theta(\iota))),\int_0^{\zeta(\iota)}g\Big(\iota,\kappa,\rho(\theta(\kappa)),\sigma(\theta(\kappa))\Big)d\kappa\Big). \quad (4.2)$$

We observe that the function $\mathcal{G}$ is continuous. Applying the assumptions (i) – (iv) we have

$$\begin{aligned}
&\Big|\mathcal{G}(\rho,\sigma)(\iota)\Big|\\
&\le\Big|f\big(\iota,h(\iota,\rho(\theta(\iota)),\sigma(\theta(\iota))),\int_0^{\zeta(\iota)}g(\iota,\kappa,\rho(\theta(\kappa)),\sigma(\theta(\kappa)))d\kappa\big)-f\big(\iota,0,0\big)\Big|+\Big|f\big(\iota,0,0\big)\Big|\\
&\le\Gamma^{-1}\Bigg[\Gamma\Big(\text{Ł}(\max\{\Big|h(\iota,\rho(\theta(\iota)),\sigma(\theta(\iota)))\Big|,\Big|\int_0^{\zeta(\iota)}g(\iota,\kappa,\rho(\theta(\kappa)),\sigma(\theta(\kappa)))d\kappa\Big|\})\Big)\\
&\quad+\Gamma\Big(\max\{\Big|h(\iota,\rho(\theta(\iota)),\sigma(\theta(\iota)))\Big|,\Big|\int_0^{\zeta(\iota)}g(\iota,\kappa,\rho(\theta(\kappa)),\sigma(\theta(\kappa)))d\kappa\Big|\}\Big)\Bigg]+\Big|f\big(\iota,0,0\big)\Big|\\
&\le\Gamma^{-1}\Bigg[\Gamma\Big(\text{Ł}(\max\{\max\{|\rho(\theta(\iota))|,|\sigma(\theta(\iota))|\}+|h(\iota,0.0)|,G\}\Big)\\
&\quad+\Gamma\Big(\max\{\max\{|\rho(\theta(\iota))|,|\sigma(\theta(\iota))|\}+|h(\iota,0.0)|,G\}\Big)\Bigg]+\Big|f\big(\iota,0,0\big)\Big|\\
&\le\Gamma^{-1}\Bigg[\Gamma\Big(\text{Ł}(\max\{\max\{||\rho||,||\sigma||\}+N,G\}\Big)+\Gamma\Big(\max\{\max\{||x||,||y||\}+N,G\}\Big)\Bigg]+M.
\end{aligned}$$

Therefore,

$$\begin{aligned}\|\mathcal{G}(\rho,\sigma)\|\le\Gamma^{-1}[&\Gamma(\text{Ł}(\max\{\max\{||\rho||,||\sigma||\}+N,G\}))\\
&+\Gamma(\max\{\max\{||\rho||,||\sigma||\}+N,G\})]+M.\end{aligned} \quad (4.3)$$

Due to inequality (4.3) and using (v), the function $\mathcal{G}$ maps $(\bar{B}_{r_0})^2$ into $\bar{B}_{r_0}$.

For the proof of continuity of G on $\bar{B}_{r_0}$ we refer the reader to [16].

Let Q_1 and Q_2 are nonempty and bounded subsets of $\bar{B}_{r_0}$, and assume that $\varepsilon>0$ is an arbitrary constant. Let $\iota_1,\iota_2\in[0,T]$, with $|\iota_1-\iota_2|\le\varepsilon$ and $\zeta(\iota_1)\le\zeta(\iota_2)$ and $(\rho,\sigma)\in Q_1\times Q_2$. Then we have

$$\begin{aligned}
&\Big|\mathcal{G}(\rho,\sigma)(\iota_2)-\mathcal{G}(\rho,\sigma)(\iota_1)\Big| \quad (4.4)\\
&\le\Big|f\big(\iota_2,h(\iota_2,\rho(\theta(\iota_2)),\sigma(\theta(\iota_2))),\int_0^{\zeta(\iota_2)}g(\iota_2,\kappa,\rho(\theta(\kappa)),\sigma(\theta(\kappa)))d\kappa\big)\\
&\quad-f\big(\iota_2,h(\iota_1,\rho(\theta(\iota_1)),\sigma(\theta(\iota_1))),\int_0^{\zeta(\iota_2)}g(\iota_2,\kappa,\rho(\theta(\kappa)),\sigma(\theta(\kappa)))d\kappa\big)\Big|
\end{aligned}$$

$$
\begin{aligned}
&+|f\big(\iota_2,h(\iota_1,\rho(\theta(\iota_1)),\sigma(\theta(\iota_1))),\int_0^{\zeta(\iota_2)}g(\iota_2,\kappa,\rho(\theta(\kappa)),\sigma(\theta(\kappa)))d\kappa\big)\\
&-f\big(\iota_1,h(\iota_1,\rho(\theta(\iota_1)),\sigma(\theta(\iota_1))),\int_0^{\zeta(\iota_2)}g(\iota_2,\kappa,\rho(\theta(\kappa)),\sigma(\theta(\kappa)))d\kappa\big)|\\
&+|f\big(\iota_1,h(\iota_1,\rho(\theta(\iota_1)),\sigma(\theta(\iota_1))),\int_0^{\zeta(\iota_2)}g(\iota_2,\kappa,\rho(\theta(\kappa)),\sigma(\theta(\kappa)))d\kappa\big)\\
&-f\big(\iota_1,h(\iota_1,\rho(\theta(\iota_1)),\sigma(\theta(\iota_1))),\int_0^{\zeta(\iota_2)}g(\iota_1,\kappa,\rho(\theta(\kappa)),\sigma(\theta(\kappa)))d\kappa\big)|\\
&+|f\big(\iota_1,h(\iota_1,\rho(\theta(\iota_1)),\sigma(\theta(\iota_1))),\int_0^{\zeta(\iota_2)}g(\iota_1,\kappa,\rho(\theta(\kappa)),\sigma(\theta(\kappa)))d\kappa\big)\\
&-f\big(\iota_1,h(\iota_1,\rho(\theta(\iota_1)),\sigma(\theta(\iota_1))),\int_0^{\zeta(\iota_1)}g(\iota_1,\kappa,\rho(\theta(\kappa)),\sigma(\theta(\kappa)))d\kappa\big)|\\
\le{}&\Gamma^{-1}\Big(\Gamma\Big(Ł\big(\{|h(\iota_2,\rho(\theta(\iota_2)),\sigma(\theta(\iota_2)))-h(\iota_1,\rho(\theta(\iota_1)),\sigma(\theta(\iota_1)))|\}\big)\Big)\\
&+\Gamma\Big(\{|h(\iota_2,\rho(\theta(\iota_2)),\sigma(\theta(\iota_2)))-h(\iota_1,\rho(\theta(\iota_1)),\sigma(\theta(\iota_1)))|\}\Big)\Big)\\
&+\omega_{r_0,\overline{G}}(f,\varepsilon)+\Gamma^{-1}\Big[\Gamma\Big(Ł\big(\Big|\int_{\zeta(\iota_1)}^{\zeta(\iota_2)}g(\iota_1,\kappa,\rho(\theta(\kappa)),\sigma(\theta(\kappa)))d\kappa\Big|\big)\Big)\\
&+\Gamma\Big(\Big|\int_{\zeta(\iota_1)}^{\zeta(\iota_2)}g(\iota_1,\kappa,\rho(\theta(\kappa)),\sigma(\theta(\kappa)))d\kappa\Big|\Big)\Big]\\
&+\Gamma^{-1}\Big[\Gamma\Big(Ł\big(\Big|\int_0^{\zeta(\iota_2)}\big|g(\iota_1,\kappa,\rho(\theta(\kappa)),\sigma(\theta(\kappa)))-g(\iota_2,\kappa,\rho(\theta(\kappa)),\sigma(\theta(\kappa)))\big|d\kappa\Big|\big)\Big)\\
&+\Gamma\Big(\Big|\int_0^{\zeta(\iota_2)}\big|g(\iota_1,\kappa,\rho(\theta(\kappa)),\sigma(\theta(\kappa)))-g(\iota_2,\kappa,\rho(\theta(\kappa)),\sigma(\theta(\kappa)))\big|d\kappa\Big|\Big)\Big]\\
\le{}&\Gamma^{-1}\Big(\Gamma\Big(Ł\big(\max\{|\iota_2-\iota_1|,|\rho(\theta(\iota_2))-\rho(\theta(\iota_1))|,|\sigma(\theta(\iota_2))-\sigma(\theta(\iota_1)))|\}\big)\Big)\\
&+\Gamma\Big(\max\{|\iota_2-\iota_1|,|\rho(\theta(\iota_2))-\rho(\theta(\iota_1))|,|\sigma(\theta(\iota_2))-\sigma(\theta(\iota_1))|\}\Big)\Big)\\
&+\omega_{r_0,\overline{G}}(f,\varepsilon)+\Gamma^{-1}\Big[\Gamma\Big(Ł\big(U_{r_0}\omega(\zeta,\varepsilon)\big)\Big)+\Gamma\Big(U_{r_0}\omega(\zeta,\varepsilon)\Big)\Big]\\
&+\Gamma^{-1}\Big[\Gamma\Big(Ł\big(\zeta_T\omega_{r_0}(g,\varepsilon)\big)\Big)+\Gamma\Big(\zeta_T\omega_{r_0}(g,\varepsilon)\Big)\Big]
\end{aligned}
$$

where

$$\omega(\zeta,\varepsilon)=\sup\{|\zeta(\iota_2)-\zeta(\iota_1)|:\iota_1,\iota_2\in[0,T],\ |\iota_2-\iota_1|\le\varepsilon\},$$

$$\zeta_T=\sup\{\zeta(\iota):\iota\in[0,T]\},$$

$$U_{r_0}=\sup\{|g(\iota,\kappa,\mu_1,\mu_2)|:\iota\in[0,T],\kappa\in[0,\zeta_T],\ \mu_1,\mu_2\in[-r_0,r_0]\},$$

$$\overline{G}=\zeta_T\sup\{|g(\iota,\kappa,\mu_1,\mu_2)|:\iota\in[0,T],\kappa\in[0,\zeta_T],\ \mu_1,\mu_2\in[-r_0,r_0]\},$$

$$H = \sup\{|h(\iota,\mu_1,\mu_2)| : \iota \in [0,T], \mu_1,\mu_2 \in [-r_0,r_0]\},$$

$$\omega_{r_0,\overline{G}}(f,\varepsilon) = \sup\{|f(\iota_2,\mu,z) - f(\iota_1,\mu,z)| : \iota_1,\iota_2 \in [0,T],$$
$$|\iota_2 - \iota_1| \le \varepsilon,\ \mu \in [-H,H],\ \ z \in [-\overline{G},\overline{G}]\}$$

and

$$\omega_{r_0}(g,\varepsilon) = \sup\{|g(\iota_2,\kappa,\mu_1,\mu_2) - g(\iota_1,\kappa,\mu_1,\mu_2)| : \iota_1,\iota_2 \in [0,T],$$
$$|\iota_2 - \iota_1| \le \varepsilon,\ \mu_1,\mu_2 \in [-r_0,r_0], \kappa \in [0,\zeta_T]\}.$$

Since (ρ,σ) was an arbitrary element of $Q_1 \times Q_2$ in (4.4), we have

$$\begin{aligned}\omega(\mathcal{G}(Q_1 \times Q_2),\varepsilon) \le \Gamma^{-1}\Big[&\Gamma\Big(Ł\big(\max\{\varepsilon,\omega(\rho,\varepsilon),\omega(\sigma,\varepsilon)\}\big)\Big)\\ &+\Gamma\Big(\max\{\varepsilon,\omega(\rho,\varepsilon),\omega(\sigma,\varepsilon)\}\Big)\Big] + \omega_{r_0,\overline{G}}(f,\varepsilon)\\ &+\Gamma^{-1}\Big[\Gamma\Big(Ł\big(U_{r_0}\omega(\zeta,\varepsilon)\big)\Big) + \Gamma\Big(U_{r_0}\omega(\zeta,\varepsilon)\Big)\Big]\\ &+\Gamma^{-1}\Big[\Gamma\Big(Ł\big(\zeta_T\omega_{r_0}(g,\varepsilon)\big)\Big) + \Gamma\Big(\zeta_T\omega_{r_0}(g,\varepsilon)\Big)\Big].\end{aligned}$$

Moreover, in the light of the uniform continuity of the functions f and g on

$$[0,T] \times [-H,H] \times [-\overline{G},\overline{G}]$$

and

$$[0,T] \times [0,\zeta_T] \times [-r_0,r_0] \times [-r_0,r_0]$$

respectively, $\omega_{r_0,\overline{G}}(f,\varepsilon) \longrightarrow 0$ and $\omega_{r_0}(g,\varepsilon) \longrightarrow 0$, as $\varepsilon \longrightarrow 0$. Also, because of the uniform continuity of ζ on $[0,T]$, $\omega(\zeta,\varepsilon) \longrightarrow 0$, as $\varepsilon \longrightarrow 0$.

Now, this remarks and the above inequality via taking the sup imply that

$$\begin{aligned}\omega(\mathcal{G}(Q_1 \times Q_2),\varepsilon) \le \Gamma^{-1}\Big[&\Gamma\Big(Ł\big(\max\{\omega(Q_1,\varepsilon),\omega(Q_2,\varepsilon)\}\big)\Big)\\ &+\Gamma\Big(\max\{\omega(Q_1,\varepsilon),\omega(Q_2,\varepsilon)\}\Big)\Big]\end{aligned}$$

which by tending $\varepsilon \to 0$ and using the continuity of Ł implies that

$$\begin{aligned}\omega_0(\mathcal{G}(Q_1 \times Q_2)) \le \Gamma^{-1}\Big[&\Gamma\Big(Ł\big(\max\{\omega_0(Q_1),\omega_0(Q_2)\}\big)\Big)\\ &+\Gamma\Big(\max\{\omega_0(Q_1),\omega_0(Q_2)\}\Big)\Big].\end{aligned}$$

Therefore,

$$\begin{aligned}\Gamma\Big(\omega_0(\mathcal{G}(Q_1 \times Q_2))\Big) \le\ &\Gamma\Big(Ł\big(\max\{\omega_0(Q_1),\omega_0(Q_2)\}\big)\Big)\\ &+\Gamma\Big(\max\{\omega_0(Q_1),\omega_0(Q_2)\}\Big).\end{aligned}$$

Thus, Theorem 4 yields that the operator $\mathcal{G}$ has a coupled fixed point. Thus, the system (4.1) has at least one solution in $(C([0,T]))^2$. □

5. Example

Example 1. Let:

$$\begin{cases} \rho(\iota) = \frac{1}{7}e^{-\iota^2} + \frac{1}{2}\dfrac{\frac{\iota}{3} + \frac{tanh(\rho(\iota)+\sigma(\iota))}{3}}{1 + \frac{\iota}{3} + \frac{tanh(\rho(\iota)+\sigma(\iota))}{3}} + \dfrac{1}{2}\dfrac{\int_0^{\iota^2} \frac{e^{-(\iota+\kappa)}sech^2(\rho(\kappa)+\sigma^2(\kappa))}{e^{(2\rho^2(\kappa)+\sigma(\kappa))}}d\kappa}{1 + \int_0^{\iota^2} \frac{e^{-(\iota+\kappa)}sech^2(\rho(\kappa)+\sigma^2(\kappa))}{e^{(2\rho^2(\kappa)+\sigma(\kappa))}}d\kappa} \\ \sigma(\iota) = \frac{1}{7}e^{-\iota^2} + \frac{1}{2}\dfrac{\frac{\iota}{3} + \frac{tanh(\rho(\iota)+\sigma(\iota))}{3}}{1 + \frac{\iota}{3} + \frac{tanh(\rho(\iota)+\sigma(\iota))}{3}} + \dfrac{1}{2}\dfrac{\int_0^{\iota^2} \frac{e^{-(\iota+\kappa)}sech^2(\sigma(\kappa)+\rho^2(\kappa))}{e^{(2\sigma^2(\kappa)+\rho(\kappa))}}d\kappa}{1 + \int_0^{\iota^2} \frac{e^{-(\iota+\kappa)}sech^2(\sigma(\kappa)+\rho^2(\kappa))}{e^{(2\sigma^2(\kappa)+\rho(\kappa))}}d\kappa}. \end{cases} \tag{5.1}$$

The above system is a case of (4.1) with

$$\theta(\iota) = \iota,\ \zeta(\iota) = \iota^2, \qquad \iota \in [0,1],$$

$$f(\iota,\rho,\sigma) = \frac{1}{7}e^{-\iota^2} + \frac{1}{2}\cdot\frac{\rho}{1+\rho} + \frac{1}{2}\cdot\frac{\sigma}{1+\sigma},$$

$$g(\iota,\kappa,\rho,\sigma) = \frac{e^{-(\iota+\kappa)}sech^2(\rho+\sigma^2)}{e^{(2\rho^2+\sigma)}},$$

and

$$h(\iota,\rho,\sigma) = \frac{\iota}{3} + \frac{\tanh(\rho+\sigma)}{3}.$$

Also, take $Ł(\iota) = \frac{2}{3}$. To prove the existence of a solution for this system, we should survey the conditions (i)-(v) of Theorem 5.

Condition (i) is clearly evident.

Now

$$\begin{aligned} |h(\iota_1,\rho,\sigma) - h(\iota_2,u,v)| &\leq \frac{|\iota_1-\iota_2|}{3} + \frac{|tanh(\rho+\sigma) - tanh(u+v)|}{3} \\ &\leq \frac{|\iota_1-\iota_2|}{3} + \frac{1}{3}(|\rho-u| + |\sigma-v|) \\ &\leq \max\{|\iota_1-\iota_2|, |\rho-u|, |\sigma-v|\} \end{aligned}$$

and

$$\begin{aligned} \left|f(\iota,\rho,\sigma) - f(\iota,u,v)\right| &= \frac{1}{2}\left|\frac{\rho}{1+\rho} + \frac{\sigma}{1+\sigma} - \frac{u}{1+u} - \frac{v}{1+v}\right| \\ &\leq \frac{1}{2}\left(\frac{|\rho-u|}{(1+\rho)(1+u)} + \frac{|\sigma-v|}{(1+\sigma)(1+v)}\right) \end{aligned}$$

$$\leq \frac{1}{2}\left(\frac{|\rho-u|}{1+\frac{1}{2}|\rho-u|}+\frac{|\sigma-v|}{1+\frac{1}{2}|\sigma-v|}\right)$$
$$\leq \frac{\max\{|\rho-u|,|\sigma-v|\}}{1+\frac{1}{2}\max\{|\rho-u|,|\sigma-v|\}}.$$

Therefore,

$$\frac{-1}{\big|f(\iota,\rho,\sigma)-f(\iota,u,v)\big|}+1 \leq \frac{-1}{2}+\frac{-1}{\max\{|\rho-u|,|\sigma-v|\}}+1.$$

Taking $\Gamma(\iota)=\frac{-1}{t}+1$, we have

$$\Gamma(Ł(\max\{|\rho-u|,|\sigma-v|\}))=\Gamma(\frac{2}{3})=\frac{-1}{2}.$$

Thus,

$$\Gamma(\big|f(\iota,\rho,\sigma)-f(\iota,u,v)\big|)\leq \Gamma(Ł(\max\{|\rho-u|,|\sigma-v|\}))+\Gamma(\max\{|\rho-u|,|\sigma-v|\}).$$

We can find that f satisfies condition (ii) of Theorem 5. Also,

$$M=\sup\{|f(\iota,0,0)|:\iota\in[0,1]\}=\sup\{\frac{1}{7}e^{-\iota^2}:\iota\in[0,1]\}=\frac{1}{7}$$

and

$$N=\sup\{|h(\iota,0,0)|:\iota\in[0,1]\}=\sup\{\frac{1}{2}e^{-\iota}:\iota\in[0,1]\}=\frac{1}{2}.$$

Moreover, g is continuous on $[0,1]\times[0,1]\times\mathbb{R}^2$.

On the other hand,

$$G=\sup\{\int_0^{\iota^2} g(\iota,\kappa,\rho,\sigma)d\kappa:\iota\in[0,1],\kappa\in[0,\iota^2]\}$$
$$\leq \sup\{|\int_0^{\iota^2} e^{-(\iota+\kappa)}d\kappa|:\iota\in[0,1],\kappa\in[0,\iota^2]\}$$
$$\leq \sup\{|e^{-\iota}(1-e^{-\iota^2})|:\iota\in[0,1]\}\leq 1.$$

Furthermore, for $r=1$,

$$\Gamma^{-1}[\Gamma(Ł(\max\{r+N,G\}))+\Gamma(\max\{r+N,G\})]+M$$
$$\leq \Gamma^{-1}[\Gamma(Ł(\max\{1+\frac{1}{2},1\}))+\Gamma(\max\{1+\frac{1}{2},1\})]+\frac{1}{7}$$
$$=\Gamma^{-1}[\Gamma(Ł(\frac{3}{2}\}))+\Gamma(\frac{3}{2})]+\frac{1}{7}=\Gamma^{-1}[\Gamma(\frac{2}{3})+\Gamma(\frac{3}{2})]+\frac{1}{7}$$
$$=\Gamma^{-1}[\frac{-1}{2}+\frac{1}{3}]+\frac{1}{7}=\Gamma^{-1}[\frac{-1}{6}]+\frac{1}{7}=\frac{6}{7}+\frac{1}{7}=1=r.$$

Consequently, all the reservations of Theorem 5 are fulfilled. Hence, the system (5.1) has at least one solution in $(C([0,1]))^2$.

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